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ON NON-NEGATIVE AND IMPROVED VARIANCE ESTIMATION FOR THE RATIO ESTIMATOR UNDER THE MIDZUNO-SEN SAMPLING SCHEME

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ABSTRACT

Various studies on variance estimation showed that it is hard to single out a best and non-negative variance estimator in finite population. This paper attempts to find improved variance estimators for the ordinary ratio estimator under the Midzuno-Sen sampling scheme. A Monte Carlo comparison has been carried out. The suggested estimator has performed well and has taken non-negative values with probability 1.

Key words: Model-based estimation, Monte Carlo Simulation, Ratio estimator, Variance estimation

1. Introduction

A major problem in a large complex survey is the selection of a variance estimation procedure. Most of the basic theory developed in the standard sampling texts deals with variance estimation for linear estimators, and, therefore, is not applicable to complex survey involving ratio and composite estimation procedures. However, these variance estimators are not free from weaknesses. Also, these estimators have not incorporated the auxiliary information.

The regression and ratio estimators are widely used in survey practice. In the past more attention has been given to the ratio estimator because of its computational ease and applicability for general sampling designs. Many variance estimators for the ratio estimator have been proposed and compared. The most of them are design-based, see, e.g., Rao (1969), Rao and Beegle (1967), Rao (1968), Rao and Rao (1971), Rao and Kuzik (1974), Royall and Eberhardt (1975), Royall and Cumberland (1978, 1981), Krewski and Chakrabarty (1981) and Wu (1982)

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among others. The theoretical comparisons of various variance estimators have been made by assuming that the variables x and y, in population, satisfy some linear regression models. Several authors studied the estimation of model-variance of ratio predictor under various models, see, e.g., Mukhopadhyay (1996) and references cited there. The issue of variance estimation, for these widely used estimators, has not been finally resolved. Studies by Wu (1982), Wu & Deng (1983) and Deng & Wu (1987) show that it is hard to single out a 'best' variance estimator; optimality depends on the performance criterion use. This article deals with the estimation of variance of the ratio estimator under the Midzuno-Sen sampling scheme when auxiliary information is available.

In section 2, we suggest variance estimator for the ratio estimator. Section 3 presents the results of a Monte Carlo study that compares the suggested estimator with the standard and available estimators. Finally, our conclusions are given in Section 4.

Let $U=\{1,...,i,...,N\}$ be a finite population and let y_i and x_i be the values of the study variable y and an auxiliary variable x for the ith population unit, $i=1,...,N$. If $A \subseteq U$, we write Σ_A for $\Sigma_{i\in A}$ and $\Sigma\Sigma_A$ for $\Sigma\Sigma_{i\neq j\in A}$. We seek to estimate the variance of the ratio estimator $\hat{Y}_R = \dfrac{\sum_s y_i}{\sum_s p_i}$ of the population total $Y=\sum_U y_i$, where $p_i = \dfrac{x_i}{X}$ with $X=\Sigma_U X_i$, ($i=1,2,...,N$), on the basis of a sample s of fixed-size n drawn according to a sampling design $p(s)$ with positive inclusion probabilities $\pi_i = P(i\in s)$ and $\pi_{ij} = P(i,j\in s)$ for every i and j. $E_P(\cdot)$ and $V_P(\cdot)$ denote the design-expectation and design-variance of an estimator. The variance of $\hat{Y}_R$, suggested by Midzuno (1950), is given by

$$V_{P1}\left(\hat{Y}_R\right) = \sum_U \Lambda(s,ii)y_i^2 + \sum\sum_U \Lambda(s,ij)y_i y_j$$

where, for $i,j=1,...,N,\ i\neq j$,

$$\Lambda(s,ij) = \left(\frac{1}{M_1}\sum_{s\ni i}\frac{1}{P_s} - 1\right) \quad \text{if } i=j$$

$$= \left(\frac{1}{M_1}\sum_{s\ni i,j}\frac{1}{P_s} - 1\right) \text{ if } i\neq j$$

and $P_s = \sum_s p_i$, $M_1 = \binom{N-1}{n-1}$.

Rao (1972) proposed

$$v_1(\hat{Y}_R) = \sum_s \Lambda(s,ii)\frac{y_i^2}{\pi_i} + \sum\sum_s \Lambda(s,ij)\frac{y_i y_j}{\pi_{ij}}$$

as an unbiased estimator of $V_{R1}(\hat{Y}_R)$. Further he stated that a sufficient condition for $v_1(\hat{Y}_R)$ to be positive is that $\Lambda(s,ij) \geq 0$ for all i, j. Chaudhuri (1975) assumes that the characteristic y can take negative values in which case the above sufficient condition is not valid. Therefore, he suggested alternative unbiased estimators by writing $V_{P1}(\hat{Y}_R)$ into different forms as

$$V_{P2}(\hat{Y}_R) = X^2\left[\sum_{i=1}^{N} t_i^2(nT_i - N) + \sum\sum_{i<j}^{N}(1-T_{ij})(t_i - t_j)^2\right],$$

$$V_{P3}(\hat{Y}_R) = \left(\frac{1}{2}\sum\sum_U Q_{ij}\right)X^2,$$

$$V_{P4}(\hat{Y}_R) = \left(\frac{1}{2}\sum\sum_U R_{ij}\right)X^2$$

where $T_i = \Lambda(s,ii)+1$, $T_{ij} = \Lambda(s,ij)+1$, $t_i = \frac{y_i}{X}$,

$$Q_{ij} = \left(\frac{T_i - 1}{N-1}t_i^2 + 2(T_{ij}-1)t_i t_j + \frac{T_j - 1}{N-1}t_j^2\right)$$

and

$$R_{ij} = \left(\frac{T_{ij}}{n-1} - \frac{1}{N-1}\right)t_i^2 + 2(T_{ij}-1)t_i t_j + \left(\frac{T_{ij}}{n-1} - \frac{1}{N-1}\right)t_j^2$$

His suggested variance estimators are then

$$v_2(\hat{Y}_R) = X^2\left[\sum_s \frac{t_i^2(nT_i - N)}{\pi_i} + \sum\sum_{i<j\in s}\frac{(1-T_{ij})(t_i - t_j)^2}{\pi_{ij}}\right]$$

$$v_3(\hat{Y}_R) = \left(\frac{1}{2} \sum\sum_s \frac{Q_{ij}}{\pi_{ij}} \right) X^2$$

$$v_4(\hat{Y}_R) = \left(\frac{1}{2} \sum\sum_s \frac{R_{ij}}{\pi_{ij}} \right) X^2$$

Chaudhuri (1976, 1981) and Chaudhuri and Arnab (1981) studied the problem of non-negative variance estimation and proposed sufficient conditions for non-negativity for their estimators

$$v_{11} = \sum \sum_{i<j \in s} C_{ij} x_i x_j \frac{(N-1)X}{(n-1)x_s} \left\{ 1 - \frac{1}{M_1} \sum_{s \ni i,j} \frac{X}{x_s} \right\}$$

$$v_{12} = v_{32} = \sum \sum_{i<j \in s} C_{ij} x_i x_j \left\{ \frac{M_1 X}{M_2 x_s} - \frac{1}{\pi_{ij} M_1} \sum_{s \ni i,j} \frac{X}{x_s} \right\}$$

$$v_{13} = \sum \sum_{i<j \in s} C_{ij} x_i x_j \left\{ \frac{X}{x_s} \left(\frac{N-1}{n-1} - \frac{1}{M_2} \sum_{s \ni i,j} \frac{X}{x_s} \right) \right\}$$

$$v_{23} = \sum \sum_{i<j \in s} C_{ij} x_i x_j \left\{ \frac{1}{\pi_{ij}} - \frac{X}{M_2 x_s} \sum_{s \ni i,j} \frac{X}{x_s} \right\}$$

where $C_{ij} = \left(\frac{y_i}{p_i} - \frac{y_j}{p_j} \right)^2$, $P(s/i) = \frac{1}{M_1 p(s)}$, $M_i = \binom{N-i}{n-i}$ and $x_s = \sum_s x_i$

However, it should be noted that none of the estimators is of the form given by Rao (1979). Moreover, these sufficient conditions cannot be satisfied either at all or except in trivial case when size measures are equal.

Rao and Vijayan (1977) suggested

$$v_{10} = v_{30} = \sum \sum_{i<j \in s} C_{ij} x_i x_j \left\{ \frac{1}{M_2 p(s)} - \frac{P(s/i)P(s/j)}{(p(s))^2} \right\}$$

and

$$v_{22} = \sum \sum_{i<j \in s} C_{ij} \frac{x_i x_j}{\Pi_{ij}} \left\{ 1 - \frac{1}{M_1} \sum_{s \ni i,j} \frac{X}{x_s} \right\},$$

which satisfy Rao's (1977) condition for the necessary form of non-negative quadratic unbiased estimators. Tracy and Mukhopadhyay (1994), also, addressed the same problem namely non-negative unbiased variance estimation for the Midzuno strategy. Vijayan et al. (1995) extended the results of Rao and Vijayan (1977) for non-negative unbiased variance estimation of quadratic forms and, in particular, discussed various estimators of variance of estimators of the population total.

2. The suggested estimator

Valliant et al. (2000) has mentioned that relatively little research has been directed toward deriving variance estimators that explicitly incorporate both design-based and model-based thinking. In this section model-design-based estimator is suggested.

Assume that $y_1, y_2, \ldots, y_N$ are exchangeable random variables having joint distribution ξ (See, Cassel et al., 1977, p.102) and that the relationship between y_i and x_i is

$$\left.\begin{aligned} & y_i = \beta x_i + \varepsilon_i \\ & \mathrm{E}_\xi(\varepsilon_i / x_i) = 0 \\ & V_\xi(\varepsilon_i / x_i) = \sigma^2 x_i^2 \\ & C_\xi(\varepsilon_i, \varepsilon_j / x_i, x_j) = \rho\sigma^2 x_i x_j \quad (i \neq j) \end{aligned}\right\} \tag{1}$$

where $\beta, \sigma^2 > 0$ and $\rho \in \left(-\frac{1}{N-1}, 1\right)$ are the parameters. Here $E_\xi(\cdot)$, $\mathrm{V}_\xi(\cdot)$ and $\mathrm{C}_\xi(\cdot)$ denote ξ-expectation, ξ-variance and ξ-covariance, respectively. The parameter ρ in the model (1) was shown to be a quite generally redundant. (See, e.g., Brewer and Tam, 1990, and Patel and Shah, 1999). Henceforth, we assume $\rho = 0$.We try to make as efficient use of auxiliary information as possible through model. To find an optimal strategy (a combination of sampling design and estimator) direct minimization of design-variance, $V_p(v)$ is impossible, but given a model ξ, we can try to minimize the anticipated variance (See, Isaki and Fuller, 1982)

$$AV(v - V_{RY}) = E_\xi E_p[(v - V_{RY})^2] - [E_\xi E_p(v - V_{RY})]^2$$

Clearly, when v is p-unbiased, the $AV(\cdot)$ becomes

$$AV(v - V_{RY}) = E_\xi E_p[(v - V_{RY})^2]$$

Here, the optimality is interpreted in sense of minimizing $E_\xi E_p[(v - V_{RY})^2]$ subject to $E_p(v) = V_{RY}$, $\forall\, Y \in R_N$.

Under model (1), we have

$$E_\xi(V_{RY}) = (\sigma^2 + \beta^2)\sum_{i\in U}\Lambda(s,ii)x_i^2 + \beta^2 \sum_{i\neq j\in U}\sum \Lambda(s,ij)x_i x_j$$

We must find v to minimize

$$E_\xi E_p[(v - V_{RY})^2] = E_p\{V_\xi(v)\} + E_p\{B_\xi(v)\}^2$$

subject to $E_\xi E_p(v) = E_\xi(V_{RY})$, where $B_\xi(v) = E_\xi(v - V_{RY})$.

Proceeding on the same line as in lemma 4.5 of Cassel et al. (1977) (See, also, Patel and Shah, 1999), it is easy to show that

$$\sigma^2 + \beta^2 \hat{=} \sum_{i\in s}\frac{y_i^2}{n x_i^2} \text{ and } \beta^2 \hat{=} \sum_{i\neq j\in s}\sum \frac{y_i y_j}{n(n-1)x_i x_j} \tag{2}$$

are, respectively, unique $p\xi$-unbiased predictors of $\sigma^2 + \beta^2$ and β^2. As *p*-unbiasedness implies $p\xi$-unbiasedness, the optimal *p*-unbiased predictor of V_{RY} is found to be

$$v_{OPT} = \sum_{i\in s}\Lambda(s,ii)\frac{y_i^2}{\pi_{i0}} + \sum_{i\neq j\in s}\sum \Lambda(s,ij)\frac{y_i y_j}{\pi_{ij0}} \tag{3}$$

where,

$$\pi_{i0} = \frac{n\Lambda(s,ii)x_i^2}{\sum_{i\in s}\Lambda(s,ii)x_i^2} \quad \text{and} \quad \pi_{ij0} = \frac{n(n-1)\Lambda(s,ij)x_i x_j}{\sum_{i\neq j\in s}\sum \Lambda(s,ij)x_i x_j}$$

It is clear from (3) that v_{OPT} will reflect the true parameter value closely when the best linear fit between y_i and x_i goes through the origin and the residual from it are small.

Remark 1. The optimal inclusion probabilities are not consistent since $\Sigma\Sigma_U \pi_{ij0} \neq (n-1)\pi_{i0}$

Remark 2. It is unlikely that a design is chosen solely for the purpose of optimum estimation of a quadratic function.

3. Simulation

The preceding variance estimators are compared empirically on 22 natural populations listed in Table 4 of Appendix B. For comparison of the variance estimators, v_1, v_{12}, v_{13}, v_{22} and v_{OPT}, a sample of size $n = 4$ was drawn using Midzuno-Sen sampling scheme from each of the populations and these variance estimators were computed. These procedure is repeated $M = 10{,}000$ times.

For each variance estimate v, its relative percentage bias is calculated as

$$RB(v) = 100 * \frac{\bar{v} - V}{V} \ ,$$

the relative efficiency as

$$RE(v) = \frac{MSE(v_1)}{MSE(v)}$$

where $\bar{v} = \frac{1}{M}\sum_{j=1}^{M} v_{(j)}$, $MSE(v) = \frac{1}{M}\sum_{j=1}^{M}(v_{(j)} - V)^2$

Table 1 reports the values of RE of the estimators v_{12}, v_{13}, v_{22}, and v_{OPT}. The values of RB(%) and probability of taking negative values are reported in Table 2 and Table 3, respectively, given in Appendix A.

Remark 3. Among v_2, v_3 and v_4 (suggested by Chaudhuri, see Section 1), v_2 is better for most cases. Moreover, the performances of v_1 and v_2 are similar. For these reasons the simulated results on these estimators are not presented here.

Remark 4. Among the estimators v_{10}, v_{20} and v_{23}, v_{10} is better than v_{23} and v_{23} is better than v_{20}. For most of the populations their RBs are negative indicating that these estimators are under estimating the true variance. Also, these estimators have taken frequently the negative values for most of the populations. Moreover, v_{10} is consistently poorer than v_{22} in both criteria. In short, the estimators v_{10}, v_{20} and v_{23} are shatteringly bad and therefore the corresponding results are omitted from the respective tables.

Table 1. RE under Midzuno Sampling

Popl.	v_1	v_{12}	v_{13}	v_{22}	v_{OPT}
1	1	84473.59	76279.33	70338.32	124200.59
2	1	63.65	66.12	65.35	89.67
3	1	3205.06	3020.11	3015.52	287.83
4	1	156.62	214.59	213.11	238.16
5	1	286.67	271.66	274.04	538.19
6	1	953.42	658.26	518.02	1735.75
7	1	249.41	310.31	318.88	413.36
8	1	187.87	254.11	257.26	178.27
9	1	8.88	12.83	13.66	14.59
10	1	123.25	147.79	133.80	209.91
11	1	103.75	130.95	129.26	199.33
12	1	48.19	54.98	54.66	96.40
13	1	1298.57	1297.42	1288.07	1419.42
14	1	65.47	98.35	97.62	128.22
15	1	18.18	17.38	17.06	21.01
16	1	42.05	36.45	30.47	11.57
17	1	201.97	183.72	177.04	300.46
18	1	29.93	24.47	20.12	37.81
19	1	22.55	20.45	16.99	28.36
20	1	9.46	8.69	7.59	15.79
21	1	24.45	27.07	24.69	47.39
22	1	67.65	62.91	56.64	45.04

Tables 1–3 lead to the following comments

- The estimators v_{12}, v_{13} and v_{22} are comparable among each other from RE point of view. Among these estimators v_{12} has smaller absolute RBs and has taken negative values for a very few populations with negligible probabilities. Overall v_{12} is the best, v_{13} the middle and v_{22} the worst for most cases.
- v_{OPT} has smaller MSE but have bigger bias (in magnitude).
- Empirically, v_{OPT} is the only non-negative estimator for all the populations.
- The scatter plot of the populations 1-8, 10-14, 17, 19 and 21 reveals that a linear model $y_i = \beta x_i + \varepsilon_i$ might be appropriate and the relationship between y and x is strong. The populations 1-8, 10-14 have the variance structure $\mathcal{V}(y_i) \propto x_i$, whereas the populations 17, 19 and 21 have the

variance structure $\mathcal{V}(y_i) \propto x_i^2$. The relationship between y and x is curvilinear for the populations 18 and 20, whereas for the populations 9, 15, 16 and 20 no systematic pattern is found though the correlation between y and x is moderate to high. Clearly, the populations 1-8, 10-14, 17, 19 and 20 have fulfilled the requirements for the v_{OPT} estimator, given in (3). Obviously, for these populations v_{OPT} has performed better than the other (except one case).

4. Conclusions

Based on the empirical study in previous section we arrive at the following conclusions.

1) We can rank the performance of v_{12}, v_{13} and v_{22} as $v_{12} \succ\succ v_{13} \succ\succ v_{22}$, where '$\succ\succ$' means 'better than', with respect to all criteria considered here. Thus, the estimators v_{12} (with $\alpha = \beta = 0$) suggested by Chaudhuri (1981) performed very well for the population having moderate to high correlation between y and x.
2) It is clear that v_{OPT} will reflect the true variance clearly when the best linear fit goes through the origin and the residual from it are small. v_{OPT} will perform badly if the true model deviates from the assumed model.

Appendix A

Table 2. RB(%) under Midzuno Sampling.

Popl.	v_1	v_{12}	v_{13}	v_{22}	v_{OPT}
1	6985.55	-11.92	-20.48	-22.46	-9.53
2	178.62	-42.77	-46.84	-48.14	-55.91
3	363.68	-14.30	-23.95	-25.64	154.60
4	248.91	-15.00	-22.18	-24.03	-15.97
5	625.18	-25.90	-33.17	-35.54	-41.40
6	-535.44	2.27	10.27	13.64	8.68
7	552.97	7.44	0.64	-1.09	-2.88
8	165.00	-5.89	-9.26	-10.08	14.22
9	55.16	-8.77	-16.99	-19.46	17.47
10	12.04	-3.38	-4.82	-5.19	-6.27
11	155.22	-10.64	-15.49	-16.87	-13.51
12	117.84	-10.40	-15.48	-17.06	-19.75
13	113.12	-1.47	-1.93	-2.05	-0.56
14	113.78	-5.59	-11.26	-12.90	-5.03
15	59.47	-30.13	-32.41	-33.01	-35.97
16	-4.92	-0.64	1.00	1.47	69.32
17	368.93	-25.47	-30.86	-32.28	-38.85
18	-55.21	-0.18	5.35	7.27	19.05
19	45.16	1.56	-5.41	-7.58	8.52
20	-14.87	6.91	9.17	9.85	4.68
21	84.07	-3.23	-8.69	-10.63	-13.82
22	92.01	-11.99	-19.64	-21.34	0.99

Table 3. Probability of taking negative values

Popl.	p_1	p_{12}	p_{13}	p_{22}	p_{OPT}
1	0.2981	0	0.0003	0.0003	0
2	0.2579	0	0.0040	0.0047	0
3	0.0032	0.0004	0.0010	0.0011	0
4	0.2549	0	0	0.0008	0
5	0.2552	0	0.0004	0.0004	0
6	0.0044	0.0001	0.0004	0.0005	0
7	0.3729	0	0	0	0
8	0.0037	0	0	0	0
9	0.2600	0.0173	0.0373	0.0376	0
10	0.4071	0	0	0	0
11	0.3343	0	0	0	0
12	0.3114	0	0	0	0
13	0.0046	0	0	0	0
14	0.3296	0	0	0	0
15	0.2379	0	0	0	0
16	0.0031	0.0001	0.0004	0.0004	0
17	0.2860	0	0	0	0
18	0.0038	0.0001	0.0003	0.0004	0
19	0.0024	0	0.0003	0.0004	0
20	0.0030	0	0.0001	0.0001	0
21	0.2923	0.0078	0.0243	0.0267	0
22	0.2876	0	0.0018	0.0018	0

Appendix B

Table 4. Study Population

Popl.	N	CV(x)	CV(y)	ρ(x,y)	Source	x	y
1	15	0.388	0.376	0.996	D. Gujarati, p.173	Money Supply	GNP
2	15	0.425	0.571	0.995	D. Gujarati, p.224	Labor input (per thousand persons)	Real gross product millions of NT ($)
3	17	0.698	0.712	0.988	Murthy(1967), p.399	area under wheat (1963)	area under wheat (1964)
4	15	0.340	0.490	0.984	D. Gujarati, p.184	Money Supply	GNP
5	20	0.480	0.346	0.980	D. Gujarati, p.227	Aerospace industry sales	Defense budget outlays
6	17	0.732	0.760	0.977	Murthy(1967), p.399	area under wheat (1963)	area under wheat (1964)
7	14	0.334	0.565	0.975	D. Gujarati, p.352	GNP	Merchandise imports
8	24	0.235	0.098	0.968	Murthy(1967), p.228	Fixed Capital	Output for Factories
9	23	0.597	0.186	0.947	D. Gujarati, p.228	Real disposable income per capita ($)	Per capita consumption of chickens (lbs)
10	14	0.276	0.217	0.944	D. Gujarati, p.352	Wage income	Consumption
11	23	0.369	0.186	0.937	D. Gujarati, p.228	Composite real price of chicken substitutes per lb weighted avg.of x2 to x5	Per capita consumption of chickens (lbs)
12	23	0.414	0.186	0.935	D. Gujarati, p.228	Real retail price of beef per lb	Per capita consumption of chickens (lbs)
13	25	0.117	0.124	0.920	Murthy(1967), p.228	Fixed Capital	Output for Factories in a region
14	23	0.390	0.186	0.912	D. Gujarati, p.228	Real retail price of pork per lb	Per capita consumption of chickens (lbs)
15	17	0.136	0.287	0.887	D. Gujarati, p.230	Long-term interest rate (%)	Nominal money crores of rupees
16	23	0.642	0.702	0.881	Murthy(1967), p.128	number of persons (1951)	no. of cultivators
17	15	0.289	0.197	0.871	D. Gujarati, p.216	Real Capital input (millions of NT,$)	Real gross product millions of NT ($)
18	25	0.607	0.657	0.868	Murthy(1967), p.128	area in sq.miles	no. of cultivators
19	17	0.607	0.712	0.853	Murthy(1967), p.399	cultivated area (1961)	area under wheat (1964)
20	19	0.524	0.647	0.829	Murthy(1967), p.128	number of persons (1961)	no. of cultivators
21	22	0.518	0.375	0.800	D. Gujarati, p.279	Income	Savings
22	13	0.217	0.392	0.787	D. Gujarati, p.203	Expected or anticipated inflation rate (%) at time t	Actual rate of inflation (in %) at time t

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